STRESS CONCENTRATION OF SINGLE AND DOUBLE NOTCHED PLATE UNDER BI-AXIAL STATE OF STRESS

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Abstract
Notches greatly increase the stress concentration in mechanical elements. The high amount of stress concentration can often lead to failure of machine parts, which can be dangerous for many parts of machine. Moreover, notches in plates cannot be avoided in design of many parts and come in non-suitable places.
This article presents an attempt to evaluate the effect of bi-axial state of stress on the stress concentration factor of flat plates with notches of different shapes such as semi circular, V-shaped and sharp-edge notches. In Addition, plates with two successive similar notches have been studied.
The computation is carried out using JL-Analyzer finite element package under plane stress conditions and under the application of tensile normal stress that generated from normal load together with shear stress.
The results show that the type of loading affects significantly the value of the stress concentration factor “k,” where its value by bi-axial state of stress is greater than that by uni-axial stress condition. Also, the notch shape and dimensions affect the stress concentration factor “k,” where the V-shape notch by notch dimension ratio “φ” of 1.00 displays the greatest value for the stress concentration factor. It is also concluded that the notch shape affects the location of the maximum stress where it takes place at the tip of the V-shape notches or at the middle point of the edge that parallel to the direction of the applied normal stress by sharp-edge notches. Also, the maximum stress locates at the point of the tangent to the semi circular notches that in the direction of the applied normal stress.

Introduction
Many researches are performed to study the factors affecting the stress concentration factor of mechanical parts. A simple method is developed for the determination of the stress concentration factor (SCP) arising from multiple, periodic, axial, elliptical holes within the wall of a gun tube. The model is based upon a simple superposition procedure with capping of any stresses that exceed yield magnitude. It is found that failure takes place at the circumference of the bore,[1].
Aluminum-alloy-sheet specimens containing various notches or fillets are tested in tension to determine their stress concentration factors in both the elastic and plastic ranges. The elastic stress concentration factors are found to be slightly higher than those calculated by Neuber's method and those obtained photoelastically by Frocht. The results show further that the stress concentration factor decreases as strains at the discontinuity enter the plastic range.[2].
Another paper deals with classical problem for cracks dislocated in a certain very specific porous elastic material. A method based upon reducing of stress concentration problem for cracks is proposed. Using Fourier integral transforms, the problem is reduced to some integral equations. For the plane-strain, the problem was treated using direct numerical treatment of a hyper-singular integral equation. In the axially symmetric case, for the penny-shaped crack, the problem is reduced to a regular Fredholm integral equation of the second order. In both cases, the calculation of the stress concentration factor is aimed where its behavior versus porosity of the material was investigated, [3]. The effect of the Poisson’s ratio on the stress concentration factor is investigated,[4]. It is found that stress concentration factor is reduced in some situations and unchanged or increased in others. Furthermore, the stress concentration factor for some mechanical elements has been considered. The complexity of these problems becomes evident especially at gear and rack pinion derives. A finite element analysis of static loading of rack-pinion drive is performed, [5]. It is concluded that the stress concentration factor varies along the rack flanks and displays its maximum value at the point of contact between the rack tooth flank and tooth root. In addition, the rack depth ratio affects significantly the stress concentration factor. The aim of this work is determining the stress concentration factor for flat plate with single and double notches. Mainly, three types of notches are considered, which are the sharp-edge, V-shape and semi circular notches.

Keywords
Stress concentration factor, notch shape, notch dimension ratio, stress ratio, free mesh technique, sharp edge notch, V-shape notch, semi circular notch.

Problem description
Finite element technique is provided to analyze a plate (100x60x10mm) with single and double notches with separation distance of 10 mm. The analysis is carried out for smooth plate of steel with modulus of elasticity 2x10^5 N/mm^2 and Poisson’s ratio 0.3. Different notch-forms are considered such as sharp-edge, V-shape and semi circular notch. Fig. 1 shows the notch forms. The ratio between the notch depth and the plate width (r/h) has the values 0.1, 0.25, 0.50 and 1.0. In addition, the specimen was subjected simultaneously to a combination of normal stress “σ” and shear stress “τ” with stress ratio “λ = τ/σ” of 0.0, 0.10, 0.15, 0.25 and 0.50.

Fig. 1 Plate and notch dimensions.

The stress concentration factor “k_t” has been calculated as the ratio between the maximum generated stress “σ_max” at the notch and the nominal principal stress for the bi-axial applied stress “σ_1”, i.e.,

\[
\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}
\]

where,

σ : The applied normal stress.
τ : The applied shear stress.
Finite element analysis using JL-Analyzer code, [6], is used to evaluate the generated elastic stresses at the specimen elements. The analysis is performed assuming plane stress condition (4-node solid element) using free mesh technique as shown in Fig.2. Several paths are taken into consideration to illustrate the influence of the notch shape, dimensions and applied stress ratio on the stress concentration factor and the location of the maximum stress.

![Fig. 2 Normal and shear stress on free meshed plate with V-shape notch.](image)

**Results and discussions**

The variation of the stress concentration factor “$k_t$” versus the notch dimension ratio “$\varphi$” for sharp-edge notch is shown in Fig. 3. It can be seen that the stress concentration factor “$k_t$” increases with the increase of the notch dimension ratio “$\varphi$”. In addition, the stress concentration factor “$k_t$” increases with the increase of the stress ratio “$\lambda$”. This means that the stress concentration factor “$k_t$” at uni-axial state of stress is smaller than that by bi-axial loading condition. For sharp-edge notch, the stress concentration factor “$k_t$” has a value of about three times that by uni-axial loading condition at “$\lambda = 0.25$” and “$\varphi = 0.1$”.

Figure 4 shows the dependence of the stress concentration factor “$k_t$” on the V-shape notch on the dimension ratio “$\varphi$” for different stress ratio “$\lambda$”. It is clear that the stress concentration factor “$k_t$” increases with the increase of both the stress ratio “$\lambda$” and the notch dimension ratio “$\lambda$”. In general, the value of the stress concentration factor “$k_t$” for V-shape notch is smaller than that by sharp-edge notch.

Also, Figure 5 illustrates the variation of the stress concentration factor “$k_t$” for the semi-circular notch. Here, the value of the stress concentration factor “$k_t$” is generally greater than that by V-shape notch and smaller than that by sharp-edge notch.

Fig.6 shows the variation of the stress concentration factor “$k_t$” for plate with double sharp-edge notches at distance of 10 mm versus the notch dimension ratio “$\varphi$” at different stress ratios. Comparison between the values of the stress concentration factor “$k_t$” for single and double sharp-edge notches shows that creation of double notches tends to increase the stress concentration factor “$k_t$” with about 3.2% for uni-axial loading condition “i.e. $\lambda = 0$” and notch dimension ratio “$\varphi$” of 0.1. However, the increase of the notch dimension ratio “$\varphi$” to reach a value of 1.0 increases the stress concentration factor “$k_t$” to 4.2%. Moreover, the increasing rate of the stress concentration factor “$k_t$” takes higher values by bi-axial state of stress where its value increases with about 1.7% for the plate with notch dimension ratio “$\varphi$” of 0.1 and 40% for plate with notch dimension ratio “$\varphi$” of 1.0 at stress ratio “$\lambda$” of 0.25.

Also, the variation of the stress concentration factor “$k_t$” for plate with double V-shape and semi-circular notches versus notch dimension ratio “$\varphi$” is shown in Figs. 7 and 8 respectively. It is clear from Figs. 6-8 that the stress concentration factor “$k_t$” reaches its maximal value for the plate with sharp-edge double notches and its minimal value for the plate with V-shape double notches.
Figure 9 shows the location of the maximum stress generated in plate with different single and double notches under uni-axial state of stress. It is clear that the stresses distributed axisymmetric where the points of maximum stress are found at the tip of the V-shape notch or at the point of the tangent parallel to the direction of the applied normal stress by the semi-circular notches. In the case of sharp-edge notches, the maximum stress takes place at the middle point of the edge that parallel to the direction of the applied normal stress.

Furthermore, the location of the maximum stress by the case of bi-axial state of stress is shown in Fig. 10. It can be noticed that the position of the point of the maximum stress is oriented according to the direction of the acting shear stress.

Figures 11 and 12 show the distribution of the stress concentration factor along the plate width for single and double notches respectively. It can be noticed that the distribution is symmetric where the maximum values take place at the notch edges.

Figure 13 and 14 illustrate the distribution of the stress concentration factor for single and double notches under the effect of bi-axial state of stress. It is clear that, the distribution is asymmetric due to the influence of the shear stress, which affects the value of the three-principle stress components that result at the notch circumference.

Conclusions
From the results obtained from the analysis of this work, the following conclusions can be drown:

1. The value of the stress concentration factor depends mainly on the number of edge of the notch where the V-shape notch (single edge) displays the minimal values and the semi circular notch (infinity number of edges) displays the maximal values. However, the sharp edge notch (two edges) displays values between them.

2. The type of loading affects significantly the value of the stress concentration factor “k”, where the value of the bi-axial state of stress is greater than that of the uni-axial stress condition.

3. The notch shape affects the location of the maximum stress where it takes place at the tip of the V-shape notches or at the middle point of the edge that parallel to the direction of the applied normal stress by sharp-edge notches. Also, the maximum stress locates at the point of the tangent to the semi circular notches that in the direction of the applied normal stress.

4. The distribution of the stress concentration factor along the plate width is symmetric by the case of uni-axial state of stress while the distribution is asymmetric by the bi-axial stress condition.

References


6. JL-analyzer version 8.0, Auto FEA Engineering Software Technology, Inc.
Fig. 3 Stress concentration factor versus notch dimension ratio for plate with single sharp-edge notch.

Fig. 4 Stress concentration factor versus notch dimension ratio for plate with single V-shape notch.

Fig. 5 Stress concentration factor versus notch dimension ratio for plate with single semi-circular notch.
Fig. 6 Stress concentration factor versus notch dimension ratio for plate with double sharp-edge notches.

Fig. 7 Stress concentration factor versus notch dimension ratio for plate with double V-shape notch.

Fig. 8 Stress concentration factor versus notch dimension ratio for plate with double semi-circular notch.
Fig. 9 Location of maximum stress for plate with single and double notches under uni-axial stress.

A - Plates with single notches.
B - Plate with double notches.

Fig. 10 Location of maximum stress for plate with single and double notches under bi-axial stress.

A - Plates with single notches.
B - Plates with double notches.
Fig. 11 Variation of stress concentration factor along the plate width for uni-axial state of stress, [ single notch, \( \varphi = 0.25 \) ].

Fig. 11 Variation of stress concentration factor along the plate width for uni-axial state of stress, [ Double notch, \( \varphi = 0.25 \) ].
Fig. 12 Distribution of stress concentration factor along the plate width for bi-axial state of stress, [single notch, $\phi = 0.25$].

Fig. 13 Variation of stress concentration factor along the plate width for bi-axial state of stress, [Double notch, $\phi = 0.25$].