DOA Estimation by Controlling the Nulls of the Antenna Array Factor

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ABSTRACT

A new method for estimating the direction-of-arrival (DOA) of unknown number of source signals is proposed. The angular directions of coherent and/or non-coherent signals are estimated by controlling the nulls of the antenna array factor. A genetic algorithm is used to find the nulls on the unit circle that minimize the array output power. The pseudo-spectrum is obtained by rotating the nulls on the unit circle and estimating the corresponding array output power. The simulation results indicate that the direction-of-arrivals are estimated accurately in the presence of a wide range of input noise levels.

KEYWORDS

Array signal processing; Direction-of-arrival (DOA); Antenna array factor; Genetic algorithm.

1. INTRODUCTION

In wireless systems, smart antennas are used for increasing the capacity where the available spectrum is very limited [1-3]. By using smart antennas, many transmitters are operating simultaneously and creating many multipath components at the receiver. Therefore, it is required that the receiver must suppress an unknown number of interfering signals which can be coherent and/or non-coherent [4-7].

In this environment, DOA estimation using antenna array processing is a very efficient technique when the propagating wave suffers from the multipath phenomena. From the various methods that have been proposed, eigen-structure methods have received wide attention because of their relatively high resolution [8]. However, all eigen-structure algorithms need exact number of sources to separate signal and noise subspaces. In most practical applications, the exact number of sources is unknown and must be estimated by any estimation method. Therefore, it is recommended to develop the DOA estimation method that does not require estimating the number of sources.

The minimum variance distortionless response (MVDR) beamformer can estimate the DOA of the source signals without knowing the number of source signals; however, it does not have good resolution compared with other methods [6]. A variation of the MUSIC algorithm is developed without using the subspace decomposition and the number of sources is not required for direction finding such as MUSIC-like method [6] and SSMUSIC (Signal Subspace Scaled MUSIC) [7]. Moreover, MUSIC algorithms use spectral searching DOA estimators where the search complexity and the estimation accuracy of these algorithms strictly depend on the number of search grid used during the search.

Because the DOA estimation is generally a computationally-expensive and highly nonlinear optimization problem, recently, the random search optimization algorithms are proposed to estimate DOA. In particular, the genetic algorithms (GA) can be used directly or with other methods to estimate the DOA of the signals [9]. Also, the modified particle swarm optimization algorithm (PSO) is proposed to achieve a global optimal solution with a fast convergence rate for code-division multiple access (CDMA) [10].

In this work, DOA is estimated by representing the array factor as a polynomial with roots located on the unit circle. When I signals are impinging on the array at the directions θ_i, $i = 1,\ldots, I$, then locating I roots of the array polynomial to coincide with the I angular locations of the received signals will reduce the output power to the noise power level. In this paper, the genetic algorithm (GA) is used to calculate the location of roots on the unit circle which minimizes the array output power. Then the pseudospectrum is obtained using the steering vectors at the directions of the root locations. The results show that the DOA and the number of signals are estimated accurately as far as the number of signals are less than the number array elements.

2. PROBLEM FORMULATION

Consider a uniform linear antenna array of $M+1$ isotropic elements on the x-axis with an inter-element spacing of $\lambda/2$. For I signals arriving from different angular directions, θ_i, each array element receives the signals plus additive white Gaussian noise. The received signal of the array m^{th} element is expressed as

$$x_m(k) = \sum_{i=1}^{I} s_i(k)\, e^{-jm\pi\sin(\theta_i)} + n_m(k), m = 0,\ldots, M \quad (1)$$

where $s_i(k)$ represents the k^{th} sample of the i^{th} signal and $n_m(k)$ represents the k^{th} noise sample of the array m^{th} element. With $\mathbf{w} = [w_1 \; w_2 \; \ldots \; w_m \; \ldots \; w_M]$ as the weight vector of the array elements, the output signal of the array is

$$y(k) = \sum_{i=1}^{I} s_i(k) \sum_{m=0}^{M} w_m\, e^{-jm\pi\sin(\theta_i)} + \sum_{m=0}^{M} w_m\, n_m(k) \quad (2)$$

When all the nulls are on the unit circle, the array factor with (M+1) weights is expressed in terms of the M nulls as

$$\sum_{m=0}^{M} w_m\, e^{-jm\pi\sin(\theta)} = \prod_{m=1}^{M} c\left(e^{-j\pi\sin(\theta)} - e^{-j\pi\sin(\phi_m)}\right) \quad (3)$$

where ϕ_m is the angle of the m^{th} null and c = w_M. Using the above equation, the output of the array can be rewritten as

$$y(k) = \sum_{i=1}^{I} s_i(k) \prod_{m=1}^{M} c\left(e^{-j\pi\sin(\theta_i)} - e^{-j\pi\sin(\phi_m)}\right) + \sum_{m=0}^{M} w_m\, n_m(k) \quad (4)$$

From equation (4), if ϕ_m is rotated on the unit circle to coincide with the angular direction of the ith signal, θ_i, then the ith signal will be eliminated from the expression of the output signal.

Assuming that the signals are ergodic random processes, the average output power of the array is estimated by a time-averaged correlation of K snapshots as

$$P_y = \mathbf{w}\mathbf{R}_{\mathbf{XX}}\mathbf{w}^H \quad (5)$$

where

$$\mathbf{R}_{\mathbf{XX}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}(k)^H \quad (6)$$

and the vectors $\mathbf{x}(k)$ and $\mathbf{w}$ contain the data samples and the weights of the antenna array elements, respectively.

In general, when the nulls of the array factor are rotated on the unit circle to coincide with the angular directions of the source signals, the output power is reduced to the value of output noise power only, i.e.,

$$P_0 = \mathbf{w}_0\mathbf{R}_{\mathbf{XX}}\mathbf{w}_0{}^H \quad (7)$$

where $\mathbf{w}_0$ is the corresponding weight vector that eliminates the signals at the array output. Assuming uncorrelated noise of zero mean and σ^2 variance, the output noise power is estimated as

$$P_0 = \mathbf{w}_0\mathbf{R}_{\mathbf{XX}}\mathbf{w}_0{}^H = \sigma^2\mathbf{w}_0\mathbf{w}_0 \quad (8)$$

Therefore, from the locations of rotated nulls on the unit circle that minimize the output power, the array factor weights, $\mathbf{w}_0$, are calculated using equation

(3). From the previous derivation, the optimization problem is formulated as follows:

$$\underset{\mathbf{w}}{\text{Minimize}} \quad \mathbf{w}\mathbf{R}_{\mathbf{XX}}\mathbf{w}^{H} \tag{9}$$

Subject to:

$$\sum_{m=0}^{M} w_m \, e^{-jm\,\pi\,\sin(\theta)} = \prod_{m=1}^{M} c\left(e^{-j\,\pi\,\sin(\theta)} - e^{-j\,\pi\,\sin(\phi_m)}\right)$$

$$\text{and} \quad \mathbf{w}\mathbf{w}^{H} = 1$$

where ϕ_m is the angle of the array factor m[th] null on the unit circle and c = w_M. The solution is obtained when the nulls of the array factor coincide with the directions of the source signals.

3. DOA ESTIMATION

The procedure for estimating DOA is explained as follows:

1. A genetic algorithm search (GA) is used to solve equation (9) and to estimate the angles of the nulls, ϕ_m' ($m = 1, 2, \ldots, M$), and the corresponding weights of the array factor, $\mathbf{w}_0$.

2. Since $\mathbf{w}_0\mathbf{w}_0^{H} = 1$, the input noise level is

$$P_n = \mathbf{w}_0\mathbf{R}_{\mathbf{XX}}\mathbf{w}_0^{H} = \sigma^2 \tag{10}$$

3. For m=1,2,...M, rotating the null by π on the unit circle $(\pi\sin(\phi_m') \Rightarrow \pi\sin(\phi_m') + \pi)$ enables computing the corresponding weight vector $\mathbf{w}_m$ using equation (3). The array output power at the estimated m^{th} null is

$$P_m' = \mathbf{w}_m\,\mathbf{R}_{\mathbf{XX}}\mathbf{w}_m^{H} \tag{11}$$

4. A pseudospectrum is obtained at the array null locations by plotting P_m versus $\hat{\theta}_m$, where

$$P_m = P_m' - P_n \tag{12}$$

When $P_m > 0$, then a signal exists and the estimated angle is

$$\hat{\theta}_m = \phi_m' \tag{13}$$

4. COMPUTER SIMULATION AND RESULT DISCUSSION

A uniform linear array of eight elements with half wave inter-element spacing is used to illustrate the effectiveness of the proposed method. Therefore, seven roots are used to estimate at most the DOA of seven source signals. The GA is used to search for the angles of arrival by minimizing the output received power. The location of the roots on the unit circle are calculated by equation (9) while the angles of arrival are estimated by using equations (10-13).

To validate the proposed method using the polynomial representation, let five source signals impinging at the array with angles -60°, -30°, 0°, 20°, and 25° when the signal-to-noise ratio (SNR) is around 10 dB for each of the five signals. Figure 1 shows the pseudospectrum obtained using the proposed method with 100 snapshots and 0 dB input noise level (σ^2). Seven power values are calculated at the estimated root location as shown in Figure 1. The real signal can be estimated by comparing its power value with the estimated noise power level ($\hat{\sigma}^2$). The estimated angles of arrival of the five signals are given in Table 1; as a result, the estimated angles using the proposed method are accurate for all possible situations including closely spaced signals with no previous assumption about the number of source signals.

Also, this method resolves the direction of the signals in a multipath channel environment. Thus, two coherent source signals impinging at the angles 20° and 30° with 10 dB SNR for each signal are simulated. Figure 2 shows the pseudospectrum of the two coherent signals with 100 snapshots where the estimated angles of the two coherent signals are 19.50° and 30.09°.

To find the effect of the signal-to-noise ratio (SNR) on the performance of estimating the DOA and the power level by controlling the roots of the array polynomial, one source signal impinging at 30° is simulated. Table 2 gives the estimated angle, $\hat{\theta}_1$, and the estimated power level, P_1, for different SNR with 100 snapshots. From Table 2, the estimated DOA of the signal is nearly insensitive to SNR variation.

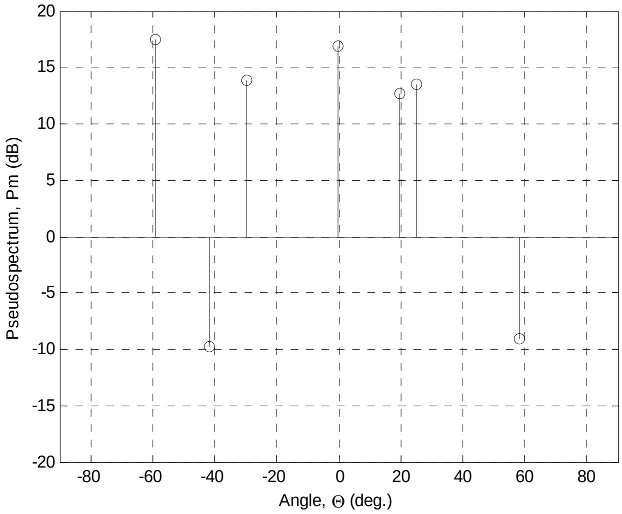

Figure 1. The pseudospectrum by controlling the nulls of the array factor when five signals are impinging at the angles -60°, -30°, 0°, 20°, and 25° with power level around 10 dB for each signal. (No. of array elements = 8, $\sigma^2 = 0\,dB$ and 100 snapshots).

Table 1. The estimated angles, $\hat{\theta}$ by controlling the nulls of the array factor for five signals. (No. of array elements = 8, $\sigma^2 = 0\,dB$ and 100 snapshots).

θ_i	P_i (dB)	$\hat{\theta}_i$
-60°	10.17	-59.30°
-30°	10.17	-29.75°
0°	9.39	-0.03°
20°	10.40	19.66°
25°	10.61	25.05°

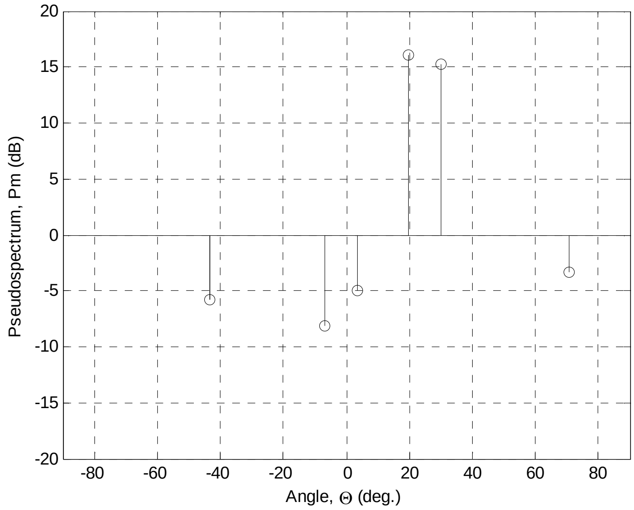

Figure 2. The pseudospectrum by controlling the nulls of the array factor when two coherent signals are impinging at the angles 20°, 30° with power level around 10 dB for each signal. (No. of array elements = 8, $\sigma^2 = 0\,dB$ and 100 snapshots).

Table 2. The estimated angle, $\hat{\theta}_1$, against SNR when one signal is impinging at 20° and with 100 snapshots. (No. of array elements = 8, $\sigma^2 = 0\,dB$ and 100 snapshots).

SNR (dB)	$\hat{\theta}_i$
10.07	19.99
8.50	19.95°
6.98	20.15°
0.11	20.14°
-3.01	20.08°
-5.22	19.02°

5. CONCLUSION

In this work, the DOA of the source signals are estimated using the array polynomial representation without knowing the number of the source signals. On the unit circle, the nulls of array polynomial are controlled to coincide with the direction of the signals by using GA search algorithm. The solution of the GA algorithm will eliminate the output power of the signals and yield the output power to the value of noise power only. A pseudospectrum is obtained using the steering vectors at the directions of the root locations which yield the minimum power and the minimum power value. From the pseudospectrum, the number of source signals and the direction of arrivals are estimated when the number of source signals is less than the number of array elements. The results indicate that the proposed method accurately estimates both the number of signals and the angle of arrivals even when the source signals are coherent.

REFERENCES

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications", IEEE J. Selec. Areas Commun. vol. 16, pp. 1451–1458, 1998.

[2] S. Anderson, M. Millnert, M. Viberg, and B. Wahlberg, "An adaptive array for mobile communication systems", IEEE Transactions on Vehicular Technology, vol. 40 (1), pp. 230 – 236, 1991.

[3] C. T. Chiang and A. C. Chang, "DOA estimation in the asynchronous DS-CDMA system", IEEE Transactions on Antennas Propagation, 51 (1), 40-47, 2003.

[4] S. N. Shahi, M. Emadi, and K. Sadeghi, "High resolution DOA estimation in fully coherent environments", Progress In Electromagnetics Research C, vol. 5, pp. 135–148, 2008.

[5] Z. C. Sha, Z. M. Liu, Z. T. Huang, and Y. Y. Zhou, "Direction Estimation of Correlated/Coherent Signals by Sparsely Representing the Signal-Subspace Eigenvectors", Progress In Electromagnetics Research C, vol. 40, pp. 37-52, 2013.

[6] Y. Zhang and B. P. Ng, "MUSIC-Like DOA estimation without estimating the number of sources", IEEE Trans. on Signal Processing, vol. 58 (3), pp. 1668-1676, 2010.

[7] Q. C. Zhou, H. T. Gao, and F. Wang, "A high resolution DOA estimation method without estimating the number of sources", Progress In Electromagnetics Research C, vol. 25, pp. 233–247, 2012.

[8] L. Godara, "Application of Antenna Arrays to Mobile Communications, Part II: Beam-Forming and Direction-of-Arrival Considerations", IEEE Proceedings of the IEEE, vol. 85 (8), pp. 1195-1245, 1997.

[9] F. Arikan, O. Koroglu, S. Fidan, O. Arikan, and M. B. Guldogan, "Multipath Separation-Direction of Arrival (MS-DOA) with Genetic Search Algorithm for HF channels", Advances in Space Research, vol. 44, pp. 641–652, 2009.

[10] J. Hung. "Modified particle swarm optimization structure approach to direction of arrival estimation", Applied Soft Computing, vol. 13, pp. 315–320, 2013.