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Article

Effect of Viscous Dissipation on Flow over a Stretching Porous Sheet Subjected to Power Law Heat Flux in Presence of Heat Source

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Abstract: The influence of viscous dissipation on laminar flow and heat transfer of a viscous incompressible fluid due to a stretching porous sheet subjected to power law heat flux in presence of heat source is studied. The equations of motion and heat transfer are transformed into a system of dimensionless non-linear ordinary differential equations which solved numerical by using shooting technique. The effects of porosity parameter, Eckert number, Prandtl number and heat source parameter on the velocity profile, temperature distribution and recovery temperature are discussed.

Keywords: viscous dissipation; heat transfer; porous medium; heat flux.

Nomenclature

u	Velocity component in x-direction	**Greek Symbols**	
v	Velocity component in y-direction	β	Heat source parameter
Ec	Eckert number	η	Pseudo similar variable
Pr	Prandtl number	λ	Permeability parameter
f	Dimensionless stream function	μ	Dynamical viscosity
CP	Specific heat at constant pressure	ρ	Density
k	Thermal conductivity	ν	Kinematical viscosity
Nu	Nusselt number	θ	Dimensionless temperature

τ	Shearing stress	x	Stream wise coordinate
T	Temperature	y	Direction normal to the plate
q_w	The local heat flux	**Subscripts**	
c	The stretching rate	w	Property at the wall
K	Permeability of the porous	∞	Free stream condition
B	constant	**Superscripts**	
c	constant	$'$	Differentiation with respect to η

1. Introduction

Boundary layer flow of viscous fluid and heat transfer on a continuously moving surface are of great importance in view of their relevance to many technological processes such that, the manufacturing of aerodynamic extrusion of plastic sheets, cooling of metallic paths rapidly, many industries like fiber, glass and polymers.

The first study of the boundary layer flow of a viscous fluid adjacent to a continuously stretching vertical sheet was considered by Sakiadis [1]. A combined experimental and analytically study of the laminar and turbulent flow and temperature distribution in the boundary layer on a continuously moving surface has been carried out by Tsou et al [2]. Erickson et al [3] and Gupta et al [4] extended Sakiadis problem, they taking into account suction or blowing of the fluid at the moving surface. Since that many researcher extended the problem in various ways. Afzal et al [5] and Banks [6] considered the power law stretching of the moving sheet. The magneto hydrodynamic flow and heat transfer have been studied by many researchers such as Abo-Eldahab [7] and Chakarabati et al [8]. Due to the several engineering applications of the flow through porous medium, such as the flow through packed beds, environmental pollution, blood theology and many other applications the extension of the problem to porous sheet attracted many researchers. Varjravelu et al [9] studied the laminar flow and heat transfer of a viscous fluid over a stretching surface with viscous dissipation. Chamkha [10] studied the problem of thermal radiation effects on MHD forced convection laminar flow adjacent to a non-isothermal wedge in the presence of a heat source or sink. Abo-Eldahab et al [11] studied the viscous dissipation and Joule heating effects on magneto hydrodynamic free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents. Jaber [12] studied the Transient magneto hydrodynamic mixed double diffusive convection along a vertical plate embedded in a non-Darcy porous medium with suction or injection. Abo-Eldahab and Elbarbary [13] examined heat and mass transfer over a vertical plate in the presence of a uniform and strong magnetic field and Hall current. Abo-Eldahab and El Aziz [14] investigated the effects of Hall current and Joule heating on electrically conducting fluid past a semi-infinite plate with strong magnetic field and heat generation/absorption. Jaber [15] studied the effect of Hall currents, and variable viscosity on free convective flow past a semi-infinite continuously stretching plate in presence of radiation. Jaber [16] studied the effect of Hall currents and variable fluid properties on magneto hydrodynamics flow past a continuously stretching vertical plate in the presence of radiation. Jaber [17] examined the effects of viscous dissipation and joule heating on magneto hydrodynamics flow of a fluid with variable properties past a stretching vertical plate. Mandal, et al [18] studied the effect of Hall current on MHD couette flow between thick arbitrarily conducting Plates in a rotating system.

Viscous dissipation effect plays an important role in natural convection in various devices which are subjected to large variations of gravitational force or which operate at high speed. Recently Barik et al [19] studied the heat and mass transfer on MHD flow through a porous medium over a stretching surface with heat source.

The present work is presented to study the viscous dissipation on laminar flow and heat transfer of a viscous fluid over a horizontal porous sheet subjected to power law heat flux in presence of heat source, also the plate is continuously stretching. Results are tabulated and presented in figures.

2. Mathematical Formulation

The present work generalizes the problem of heat transfer over a stretching porous sheet subjected to power law het flux in presence of heat source which presented by Hitesh Kumar [20].Consider the steady two dimensional flow of a viscous fluid over a horizontal plate coinciding with the plane y = 0. The plate is subjected to a power law heat flux in presence of heat source. The plate is continuously stretching by two equal powers with opposite directions. The speed is assumed to be proportion to its distance from the slit point. The flow is considered to be in the x-direction which is taken along the plate, see fig. 1.

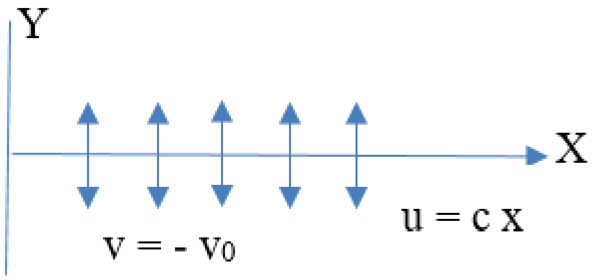

Figure 1. Physical coordinate system

Under these assumptions and taking into account the Boussinesq approximation, the boundary layer laminar flow is governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \tag{1}$$

$$u\,\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\upsilon}{K}u \tag{2}$$

$$u\,\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_P}\,\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho C_P}(T_\infty-T)-\frac{\upsilon}{C_P}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The physical problem suggested the following initial and boundary conditions:

$$\begin{aligned} &at\quad y=0:\ u=cx,\quad v=-v_0,\quad \frac{\partial T}{\partial y}=B\,x^2\\ &at\quad y\to\infty:\ u=v=0,\quad T=T_\infty \end{aligned} \tag{4}$$

where c is the stretching rate which considered to be a positive integer.

The continuity equation suggested the solution

$$u=c\,x\,f'(\eta),\quad v=-\sqrt{\upsilon c}\ f(\eta),\ \eta=\sqrt{c\upsilon^{-1}}\ y,\ T=T_\infty+Bx^2\sqrt{\upsilon c^{-1}}\ \theta(\eta)$$

Using these dimensionless parameters the governing equations, eq. 2 and 3 can be reduced to the following system of ordinary differential equations

$$f'''+f\ f''-f'^{\,2}-\lambda f'=0 \tag{5}$$

$$\theta''+\Pr f\,\theta'-2\Pr f'\theta-\beta\theta+\Pr Ec\,f''^2=0 \tag{6}$$

The boundary conditions are transformed to:

At the surface of the plate $f'(0)=1,\ \ f(0)=1,\ \ \theta'(0)=1$

Also, as $\eta\to\infty: f'=f=\theta=0$,

where $\Pr=\dfrac{\rho\upsilon C_P}{k}$ is the Prandtl number, $Ec=\dfrac{c^{\frac{5}{2}}}{BC_P\sqrt{\upsilon}}$ is the Eckert number, $\lambda=\dfrac{\upsilon}{c\,k}$ is the porosity parameter.

The physical quantities of interest Nusselt number and the local shear stress τ_w are defined as

$$q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Thus the Nusselt number is given by

$$Nu=\frac{q_w\,x}{k}=-B\,x^3\,\theta'(0)$$

The wall shear stress is defined as

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu\, x c^{\frac{3}{2}}}{\sqrt{\upsilon}} f''(0)$$

The numerical values of $f'(0)$ and $\theta'(0)$ for Pr=0.72, and several values of the Eckert and others parameters are tabulated in table 1.

3. Results and Discussion

The transformed governing system consists of two non-linear ordinary differential equations, namely equations 5 and 6. They solved numerically by using the fourth order Runge-Kutta method with the shooting technique. The effect of all the involved parameters on the velocity and temperature profiles, the local Nusselt number and the shear stress is presented and discussed. The velocity and heat profiles for involved parameters are shown in Figures 2 – 6. Figs. 2 and 3 present the effect of permeability parameter λ on velocity and temperature profiles, the increasing of permeability parameter λ causes decreasing in the velocity and give rise to increase the temperature profiles. Fig. 4 shows that the increasing of Eckert number leads to increase the temperature while temperature profile decreases as the heat source parameter increases as shown in Fig. 5. Finally, Fig.6 investigates the effect of the Prandtl number, the increase in the Prandtl number causes markedly increase in the initial magnitude of the temperature in the boundary layer, while this effect takes the reverse order as fluid go far from the plate. Table 1 shows that the local skin friction for the flow decreases as the porosity parameter λ increases, while other parameters do not affect the local skin friction. . Also, it is easily seen that the increased values of $\Pr, Ec, \lambda$ parameters increase the local Nusselt number, while it is decreased by increasing the heat source parameter β .

Table 1. Variation of the dimensionless skin friction and local Nusselt number for various parameters

Pr	**Ec**	**λ**	**β**	**-f '' (0)**	**θ ' (0)**
0.72	3	1	1	2.00007	0.395499
2	3	1	1	2.00007	1.08867
5	3	1	1	2.00007	1.50712
0.72	4	1	1	2.00007	1.54928
0.72	5	1	1	2.00007	2.60963
0.72	3	3	1	2.56155	0.735769
0.72	3	5	1	3	1.01252
0.72	3	1	3	2.00007	0.219914
0.72	3	1	5	2.00007	0.145518

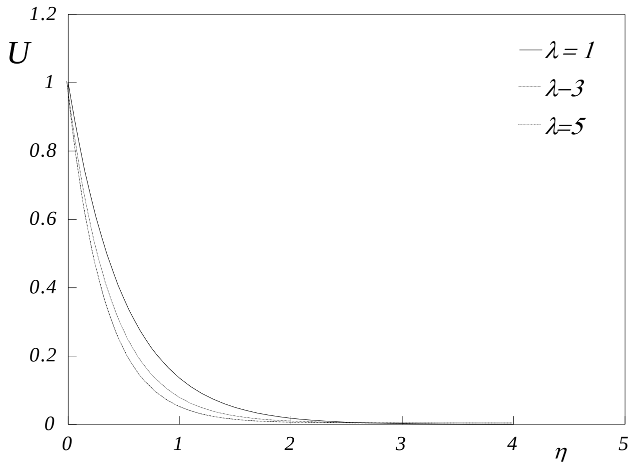

Fig. 2. Variation of dimensionless velocity f' for various values of permeability parameter λ with Pr = 0.72, β = 1, Ec = 3.

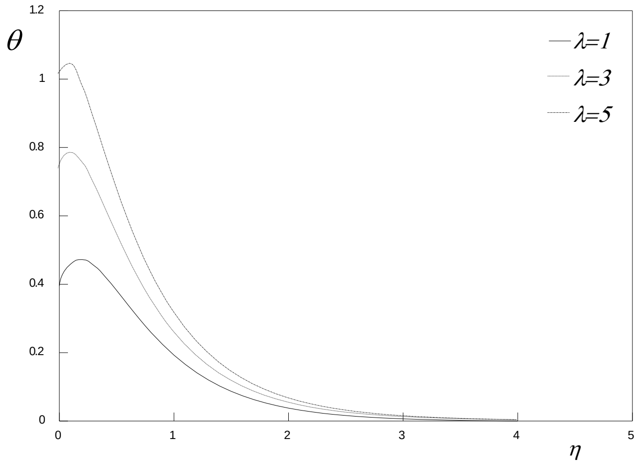

Fig. 3. Variation of dimensionless temperature θ for various values of permeability parameter λ with Pr = 0.72, β = 1, Ec = 3.

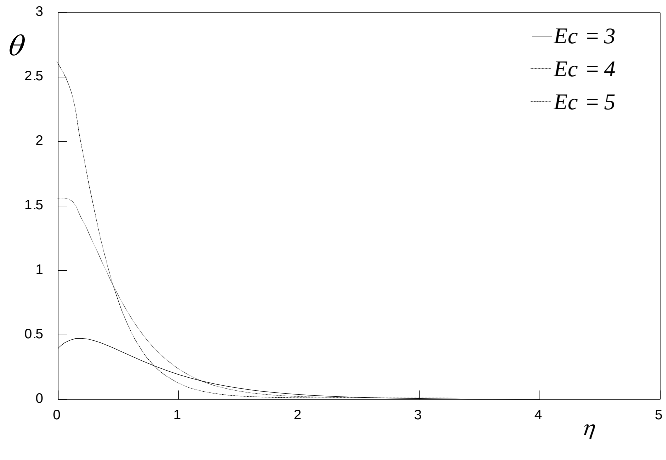

Fig. 4. Variation of dimensionless temperature θ for various values of Eckert parameter Ec with Pr = 0.72, β = 1, λ = 1.

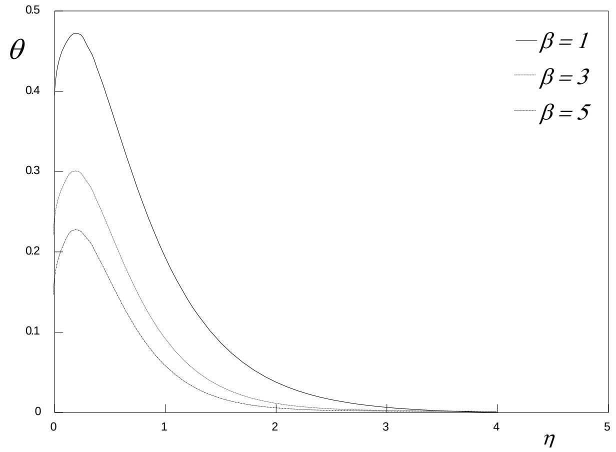

Fig. 5. Variation of dimensionless temperature θ for various values of the heat source parameter β with Pr = 0.72, Ec= 3, λ = 1.

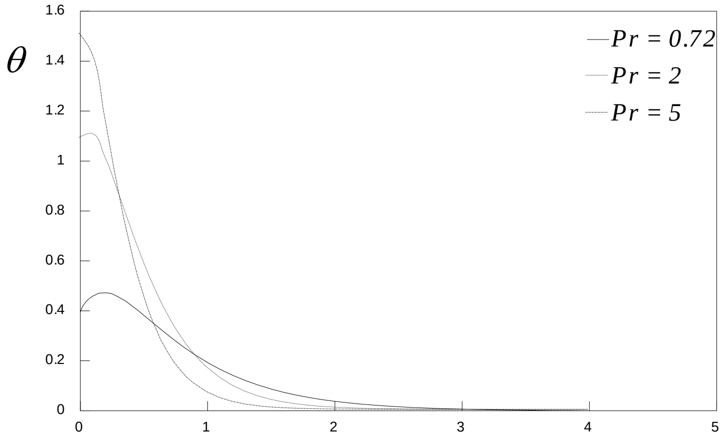

Fig. 6. Variation of dimensionless temperature θ for various values of the Prandtl number Pr with Ec = 3, β = 1, λ = 1.

4. Conclusions

The problem of steady, laminar, convection heat transfer over a horizontal, permeable and continuously stretching plate with a power-law heat source is investigated. The governing equations are transformed into a system of ordinary differential equations that solved numerically. The present study shows that:

I. Increasing the porosity parameter λ increases the temperature and the local Nusselt number and decreasing the velocity and the local skin friction.

II. Increasing the Eckert number increases both the local Nusselt number and temperature profiles.

III. Increasing the Prandtl number Pr increases both the initial values of temperature and the local Nusselt number.

IV. As the heat source parameter β increases, the temperature and local skin friction decrease.

V. No effect of Ec, Pr and heat source parameter β on velocity profiles so no figures are presented herein to limit the figures number, also they do not affect the local skin friction.

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